Lifting-Surface Theory for an Oscillating T-Tail

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A lifting-surface theory for predicting aerodynamic forces on an oscillating T-tail is presented with special reference to the effects of tailplane dihedral and tailplane incidence. The analysis is confined to the incompressible flow case. By introducing a new coordinate system oscillating coincidentally with the in-plane motion of the tailplane and by employing a perturbation technique, the boundary-value problems are derived. The integral equations for the prediction of the antisymmetric load distributions on the tailplane, which are induced by the in-plane motion of the tailplane, are derived from the second-order boundary-value problem, while the first-order problems are those of conventional lifting-surface theories. A method for solving the integral equations is proposed and the working forms of them are given. Some numerical examples for simplified T-tails are given and compared with the experimental results. The theory is useful for predicting the effects of tailplane dihedral and tailplane incidence on the flutter speed of a T-tail.

Nomenclature

A = aspect ratio
a = nondimensional axis of yaw of tailplane (positive toward leading edge)
b_0 = semichord at junction
b = local semichord
C_p = $(p-p_\infty)/(1/2\rho U_0^2)$
h_s = z-coordinate of surface of tailplane
h_f = y-coordinate of surface of fin
h_f' = y'-coordinate of surface of fin
H = nondimensional sideslip displacement of tailplane ($H = H_0 e^{ikt}$)
k = reduced frequency $[(b_0\omega)/U_0]$
l_s = semispan of tailplane
l_f = span of fin
p = pressure
Δp = pressure difference ($\Delta p = \overline{\Delta p}\, e^{ikt}$)
q_j = dimensionless generalized coordinate of jth mode ($q_j = \bar{q}_j e^{ikt}$)
S_s, S_f = l_s/b_0, l_f/b_0
T = time
t = dimensionless time
U, V, W = perturbation velocities
U_0 = freestream velocity
u, v, w = dimensionless perturbation velocities
X, Y, Z; x, y, z = Cartesian coordinates of a system fixed to freestream
x', y', z'; ξ', η', ζ' = Cartesian coordinates of a system whose motion is coincident with yawing and sideslip motion of tailplane
x_s^* = $(x'-\xi_{sm})/(b_s/b_0)$
x_f^* = $(x'-\xi_{fm})/(b_f/b_0)$
$\xi_{sl}, \xi_{st}, \xi_{sm}$ = x'-coordinate of leading edge, trailing edge, and midchord line of tailplane, respectively
$\xi_{fl}, \xi_{ft}, \xi_{fm}$ = x'-coordinate of leading edge, trailing edge, and midchord line of fin, respectively
y^* = y'/S_s
η^* = η'/S_s
z^* = z'/S_f
ζ^* = ζ'/S_f
α = angle of attack of tailplane
Γ = dihedral angle of tailplane
Λ = sweep angle of quarter-chord line
λ = taper ratio
ρ = air density
ϕ' = perturbation potential
ϕ = dimensionless perturbation potential
Ψ = yawing angle of tailplane ($\Psi = \Psi_0 e^{ikt}$)
ω = angular frequency

Superscripts

$\dot{}$ = d/dt
(s) = symmetric (even function of y')
(as) = antisymmetric (odd function of y')
(1), (2) = first- and second-order, respectively
$\bar{}$ = amplitude of sinusoidal oscillation

Subscripts

1, 2 = first-order and second-order, respectively
s = tailplane
f = fin

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I. Introduction

It has been pointed out in several papers[1-3,14] that the flutter behavior of a T-tail is critically dependent on the angle of attack and the dihedral angle of the tailplane. The change in dihedral due to tailplane bending caused by the trim load[1,15] will also influence the flutter speed. It is, therefore, vital to take into account these phenomena in designing a T-tail since in practice the incidence of the tailplane (or the trim tail load) is rarely zero.

To these phenomena, the aerodynamic rolling moments due to the in-plane motions of the tailplane are responsible.[4] The other rotary effect, viz., the yawing moment due to rolling motion of the tailplane may have some influence on the T-tail flutter,[16] but its nature and relative importance have not yet been known. The present paper is concerned only with the former, viz., the rolling moment due to the in-plane motion of the tailplane.

The existing methods for estimating these forces, are approximate ones which utilize strip theory or steady lifting-line theory[17] in the quasi-steady manner. They, therefore, cannot avoid an uncertainty with regard to 1) three-dimensional effect, 2) unsteady effect and 3) the influence of the fin.

For the more accurate prediction of these forces, a new unsteady lifting-surface theory, in which the effect of the in-plane motion of the tailplane is properly taken into account, should be developed since the conventional lifting-surface theories[6-8] of a T-tail afford no information about this point.

In the recent papers,[9-10] Ichikawa and Isogai developed a lifting-surface theory which accounts for the effect of the in-plane motion of the tailplane. In Ref. 9, both the problems of an isolated wing and its extension to the complete T-tail configuration were treated from the vortex theory standpoint and the physical aspects of the problem were made clear, but without giving the method for the practical calculation. In Ref. 10, a kernel function procedure, which is convenient for the practical calculation, was developed for an isolated wing and a possibility of extending the theory for general subsonic flow also was shown.

This paper presents a kernel function procedure for a complete T-tail configuration, which is the extension of the method developed in Ref. 10.

In view of the fact that the problem of the symmetric mode of vibration is not different from that of a conventional isolated wing, the T-tail is assumed to be oscillating harmonically in an antisymmetric mode.[9] The fin is, therefore, oscillating normal to its surface about zero mean angle of attack while the tailplane is oscillating normal to its surface in an antisymmetric mode as well as in the in-plane motions (sideslip and yaw).

The techniques developed in Ref. 9 and Ref. 10, namely the introduction of a moving coordinate system, whose motion is coincident with the in-plane motion of the tailplane, and the employment of a perturbation technique, also are essential in the present theory.

It would be worth mentioning that the present theory is capable of treating either positive or negative† values of incidence (or geometrical dihedral) of the tailplane as well as the change in dihedral due to the tailplane bending by the trim load.

Although the compressibility effect may be important, the present analysis is restricted to the incompressible flow case because an attempt to extend the present theory to general subsonic flow will encounter a great difficulty.[10]

II. Basic Equations

A diagram of the T-tail under consideration is given in Fig. 1. The Cartesian co-ordinate system (X, Y, Z) is fixed to the undisturbed mainstream of a velocity, U_0. The positive direction of X is that of the mainstream. The tailplane is located near the XY-plane. The fin is located near the XZ-plane. They are so thin that we can neglect the thickness effect.

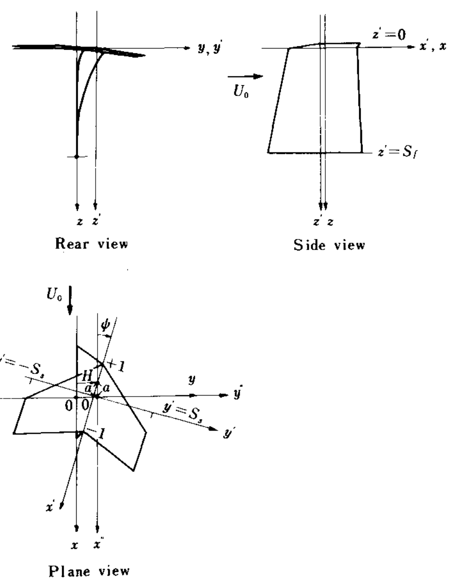

Fig. 1 Definitions of coordinate systems.

† A negative incidence or negative dihedral usually gives a favorable effect on a T-tail flutter (the flutter speed increases).

In addition to the vibration normal to the surface about the mean position, the tailplane is conducting in-plane motion (the yawing and sideslip oscillation) which is induced by the out of plane motion of the fin. The origin of the XYZ-coordinate system is at the mean position of the midchord of the midspan of the tailplane. It is reasonable to assume that the line of the junction of the tailplane and fin is always straight.

The T-tail is submerged in incompressible nonviscous fluid. We can, therefore, assume the existence of the perturbation potential $\phi'(X, Y, Z, T)$. For later convenience, the physical quantities are nondimensionalized by the root semichord b_0 and U_0 as follows:

$$x = X/b_0, \quad y = Y/b_0, \quad z = Z/b_0, \quad t = (U_0/b_0)T$$
$$\phi(x, y, z, t) = \phi'(X, Y, Z, T)/(U_0 b_0) \tag{1}$$
$$u = U/U_0, \quad v = V/U_0, \quad w = W/U_0$$

The nondimensional perturbation velocities u, v and w have the following relations with the nondimensional perturbation potential ϕ:

$$u = \phi_x, \quad v = \phi_y, \quad w = \phi_z \tag{2}$$

The equation governing ϕ is

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{3}$$

From the Bernoulli equation, the pressure coefficient becomes

$$C_p = -2\phi_x - 2\phi_t - \phi_x{}^2 - \phi_y{}^2 - \phi_z{}^2 \tag{4}$$

The exact tangency conditions on the upper and the lower surfaces of the tailplane and fin are

$$\phi_z = \partial h_s/\partial t + (1+\phi_x)\,\partial h_s/\partial x + \phi_y\,\partial h_s/\partial y \quad \text{on} \quad z = h_s \pm 0 \tag{5}$$
$$\phi_y = \partial h_f/\partial t + (1+\phi_x)\,\partial h_f/\partial x + \phi_z\,\partial h_f/\partial z \quad \text{on} \quad y = h_f \pm 0 \tag{6}$$

respectively, where $h_s(x, y, t)$ is the z-coordinate of the surface of the tailplane and $h_f(x, z, t)$ is the y-coordinate of the fin.

Now we introduce a new coordinate system (x', y', z') whose motion is exactly coincident with the yawing and sideslip oscillation of the tailplane (see Fig. 1). When we denote the yawing angle and sideslip displacement by Ψ and H, respectively, the relations between the two coordinate systems can be obtained as follows:

$$x' = x - \Psi y, \quad y' = \Psi(x+a) + (y-H), \quad z' = z \tag{7}$$

where the higher order terms of Ψ and H are neglected.‡

In the new coordinate system, Eq. (3) becomes

$$\phi_{x'x'} + \phi_{y'y'} + \phi_{z'z'} = 0 \tag{8}$$

From Eq. (4) we have

$$C_p = -2[\phi_t - \dot{\Psi} y' \phi_{x'} + \dot{\Psi}(x'+a)\phi_{y'} + \phi_{x'} + \Psi\phi_{y'} - \dot{H}\phi_{y'}] - \phi_{x'}{}^2 - \phi_{y'}{}^2 - \phi_{z'}{}^2 \tag{9}$$

by neglecting the higher order terms like $\Psi\phi_{x'}\phi_{y'}$, $\dot{H}\phi_{x'}\phi_{y'}$, etc.‡ From Eq. (5) we have

$$\phi_{z'} = [\partial/\partial t - \dot{\Psi} y'\,\partial/\partial x' + \dot{\Psi}(x'+a)\,\partial/\partial y']h_s + (\partial/\partial x' + \Psi\,\partial/\partial y')h_s - \dot{H}\,\partial h_s/\partial y' + \phi_{x'}\,\partial h_s/\partial x' + \phi_{y'}\,\partial h_s/\partial y' \quad \text{on} \quad z' = h_s \pm 0 \tag{10}$$

by neglecting the higher order terms like $\Psi^2 \phi_{y'}\,\partial h_s/\partial y'$, $\Psi^2 \phi_{x'}\,\partial h_s/\partial x'$, etc.‡ From Eq. (6) we have

$$\phi_{y'} = \Psi\phi_{x'} + \partial h_f/\partial t + (1+\phi_{x'})\,\partial h_f/\partial x' + \phi_{z'}\,\partial h_f/\partial z' \quad \text{on} \quad y' = h_f' \pm 0 \tag{11}$$

by neglecting the higher order term $\Psi\phi_{y'}\,\partial h_f/\partial x'$,‡ where h_f' is the y'-coordinate of the surface of the fin. The relation between h_f and h_f' is obtained from Eq. (7) as

$$h_f = -\Psi(x'+a) + h_f' + H \tag{12}$$

h_s and h_f can be expressed in the new coordinate system as

$$h_s(x', y', t) = h_{s0}(x', y') + \sum_j \bar{h}_{sj}(x', y') q_j(t) \tag{13}$$
$$h_f(x', z', t) = \sum_j \bar{h}_{fj}(x', z') q_j(t) \tag{14}$$

‡ It can be shown that the higher order terms neglected in Eqs. (7 and 9–11) do not contribute to the first- and second-order boundary-value problems, in which we are interested in the ensuing analysis.

where $h_{s0}(x', y')$ is the steady component and may be the function of the angle of attack α, the dihedral angle Γ and the geometrical twist of the tailplane. The change in dihedral under the trim tail load can be included in h_{s0} also. $\bar{h}_{sj}(x', y')$ and $\bar{h}_{fj}(x', z')$ give the deflection of the tailplane and fin, respectively, in the jth mode of vibration. $q_j(t)$ is the generalized coordinate of the jth mode. Further, we assume that $\bar{h}_{sj}(x', y')$ is an odd function of y' because we are only interested in the antisymmetric mode of vibration. Ψ and H have the following relations with q_j because the junction line is assumed to be straight.

$$\Psi = -\sum_j [\partial \bar{h}_{fj}/\partial x' q_j]_{z'=0} \tag{15}$$

$$H = \sum_j \bar{h}_{fj}(-a, 0)q_j \tag{16}$$

Hereafter we will consider only the jth mode of vibration and drop the suffix j.

Although the boundary-value problem is completed by adding the other boundary conditions of the finiteness of ϕ, zero pressure difference of Δp_s across the plane $z' = 0$ at every point (x', y') off the tailplane, zero pressure difference of Δp_f across the plane $y' = 0$ at every point (x', z') off the fin, and the Kutta condition, it is not possible to solve the problem exactly. A perturbation technique is, therefore, introduced. It is assumed that h_{s0}, Ψ, H and q are all small quantities of the same order, namely $h_{s0} = 0(\varepsilon)$, $\Psi = 0(\varepsilon)$, $H = 0(\varepsilon)$ and $q = 0(\varepsilon)$. The following expansion of ϕ is assumed

$$\phi = \phi_1 + \phi_2 + \phi_3 + \cdots \tag{17}$$

where ϕ_1 is the order of ε, ϕ_2 is the order of $\varepsilon^2, \ldots$ etc. The pressure differences

$$\Delta p_s = (1/2)\rho U_0{}^2[C_p(x', y', h_s - 0, t) - C_p(x', y', h_s + 0, t)] \tag{18}$$

on the tailplane and

$$\Delta p_f = (1/2)\rho U_0{}^2[C_p(x', h_f' - 0, z', t) - C_p(x', h_f' + 0, z', t)] \tag{19}$$

on the fin are also assumed to be expanded as

$$\Delta p_s = \Delta p_{s1} + \Delta p_{s2} + \Delta p_{s3} + \cdots \tag{20}$$

$$\Delta p_f = \Delta p_{f1} + \Delta p_{f2} + \Delta p_{f3} + \cdots \tag{21}$$

where the same order as the ϕ_i $(i = 1, 2, 3, \ldots)$ is assumed for $\Delta p_{si}/(2\rho U_0{}^2)$ and $\Delta p_{fi}/(2\rho U_0{}^2)$.

Before substituting Eq. (17) into Eqs. (8–11), it is desirable to transform Eqs. (10) and (11) so that the normal wash conditions are given on $z' = \pm 0$ plane and $y' = \pm 0$ plane, respectively. Equation (9) is also transformed so that the pressure differences Δp_s and Δp_f are given on the plane $z' = 0$ and on the plane $y' = 0$, respectively. These can be accomplished by expanding $\phi(x', y', h_s \pm 0, t)$ in Taylor series about $z' = \pm 0$ and by expanding $\phi(x', h_f' \pm 0, z', t)$ in Taylor series about $y' = \pm 0$.

After completing the substitution, following the standard procedure of the perturbation technique with the aid of Eqs. (18–21), we obtain basic equations for the boundary-value problems corresponding to the first and the second order.

1. Equations of the Order of ε

It is convenient to write

$$\phi_1 = \phi_1{}^{(s)} + \phi_1{}^{(as)} \tag{22}$$

$$\Delta p_{s1} = \Delta p_{s1}{}^{(s)} + \Delta p_{s1}{}^{(as)} \tag{23}$$

where $\phi_1{}^{(s)}$ and $\Delta p_{s1}{}^{(s)}$ are even functions of y' while $\phi_1{}^{(as)}$ and $\Delta p_{s1}{}^{(as)}$ are odd functions of y'. The problems for $\phi_1{}^{(s)}$ and $\phi_1{}^{(as)}$ can be treated separately.

a) Equations for $\phi_1{}^{(s)}$

$$\phi_{1x'x'}{}^{(s)} + \phi_{1y'y'}{}^{(s)} + \phi_{1z'z'}{}^{(s)} = 0 \tag{24}$$

$$\Delta p_{s1}{}^{(s)} = 2\rho U_0{}^2\, \partial(\phi_1{}^{(s)*})/\partial x' \tag{25}$$

where $\phi_1{}^{(s)*}$ is defined by

$$\phi_1{}^{(s)*} = [\phi_1{}^{(s)}(x', y', +0) - \phi_1{}^{(s)}(x', y', -0)]/2 \tag{26}$$

$$\phi_{1z'}{}^{(s)} = \partial h_{s0}/\partial x' \quad \text{on} \quad z' = 0 \tag{27}$$

It should be noticed that the pressure difference across the plane $y' = 0$ and $\phi_{1y'}{}^{(s)}$ on $y' = 0$ are always zero.

b. Equations for $\phi_1{}^{(as)}$

We observe that the boundary-value problem for $\phi_1{}^{(as)}$ is linear in q. Assuming sinusoidal motion, utilizing complex quantities, writing

$$\phi_1{}^{(as)} = \bar{\phi}_1{}^{(as)} e^{ikt}, \qquad q = \bar{q}\, e^{ikt}, \qquad \Delta p_{s1}{}^{(as)} = \overline{\Delta p}_{s1}{}^{(as)} e^{ikt}$$

and

$$\Delta p_{f1} = \overline{\Delta p}_{f1}\, e^{ikt}$$

we obtain

$$\bar{\phi}_{1x'x'}{}^{(as)} + \bar{\phi}_{1y'y'}{}^{(as)} + \bar{\phi}_{1z'z'}{}^{(as)} = 0 \tag{28}$$

$$\overline{\Delta p}_{s1}{}^{(as)} = 2\rho U_0{}^2(ik + \partial/\partial x')\bar{\phi}_{s1}{}^{(as)*} \tag{29}$$

$$\overline{\Delta P}_{f1} = 2\rho U_0{}^2(ik + \partial/\partial x')\bar{\phi}_{f1}{}^{(as)*} \tag{30}$$

In Eqs. (29) and (30), $\bar{\phi}_{s1}{}^{(as)*}$ and $\bar{\phi}_{f1}{}^{(as)*}$ are defined by

$$\bar{\phi}_{s1}{}^{(as)*} = [\bar{\phi}_1{}^{(as)}(x', y', +0) - \bar{\phi}_1{}^{(as)}(x', y', -0)]/2 \tag{31}$$

$$\bar{\phi}_{f1}{}^{(as)*} = [\bar{\phi}_1{}^{(as)}(x', +0, z') - \bar{\phi}_1{}^{(as)}(x', -0, z')]/2 \tag{32}$$

The tangency conditions become

$$\bar{\phi}_{1z'}{}^{(as)} = (ik + \partial/\partial x')\bar{h}_s \bar{q} \quad \text{on} \quad z' = 0 \tag{33}$$

$$\bar{\phi}_{1y'}{}^{(as)} = (ik + \partial/\partial x')\bar{h}_f \bar{q} \quad \text{on} \quad y' = 0 \tag{34}$$

Among the boundary-value problems derived above, the problem for $\phi_1{}^{(s)}$ is no more than that of a rigid isolated wing in steady flight without yaw and sideslip. The problem for $\bar{\phi}_1{}^{(as)}$ has the same form as that which has already been treated by several authors.[6–8] It is clear from Eqs. (24–34) that the effect of yaw and sideslip of the tailplane does not appear in the boundary-value problems of this order.

2. Equations of the Order of ε^2

It is also convenient to write

$$\phi_2 = \phi_2{}^{(s)} + \phi_2{}^{(as)} \tag{35}$$

$$\Delta p_{s2} = \Delta p_{s2}{}^{(s)} + \Delta p_{s2}{}^{(as)} \tag{36}$$

where $\phi_2{}^{(s)}$ and $\Delta p_{s2}{}^{(s)}$ are even functions of y' while $\phi_2{}^{(as)}$ and $\Delta p_{s2}{}^{(as)}$ are odd functions of y'. The second-order symmetric problem of $\phi_2{}^{(s)}$, however, does not seem to be important in the problem of the T-tail flutter.§ We will, therefore, pursue only the antisymmetric problem of $\phi_2{}^{(as)}$.¶

We observe that the boundary-value problem for $\phi_2{}^{(as)}$ is still linear in Ψ and H or q. We can, therefore, assume sinusoidal variation of $\phi_2{}^{(as)}$, $\Delta p_{s2}{}^{(as)}$ and Δp_{f2}, utilize complex quantities and write $\phi_2{}^{(as)} = \bar{\phi}_2{}^{(as)} e^{ikt}$, $\Delta p_{s2}{}^{(as)} = \overline{\Delta p}_{s2}{}^{(as)} e^{ikt}$ and $\Delta p_{f2} = \overline{\Delta p}_{f2}\, e^{ikt}$. Then the equations for $\bar{\phi}_2{}^{(as)}$ become

$$\bar{\phi}_{2x'x'}{}^{(as)} + \bar{\phi}_{2y'y'}{}^{(as)} + \bar{\phi}_{2z'z'}{}^{(as)} = 0 \tag{37}$$

The pressure difference across the plane $z' = 0$ is given by

$$\overline{\Delta p}_{s2}{}^{(as)} + 2\rho U_0{}^2 \Phi_s{}^{(as)} = 2\rho U_0{}^2(ik + \partial/\partial x')\bar{\phi}_{s2}{}^{(as)*} \tag{38}$$

where $\bar{\phi}_{s2}{}^{(as)*}$ is defined by

$$\bar{\phi}_{s2}{}^{(as)*} = [\bar{\phi}_2{}^{(as)}(x', y', +0) - \bar{\phi}_2{}^{(as)}(x', y', -0)]/2 \tag{39}$$

and where

$$\begin{aligned}\Phi_s{}^{(as)}(x', y') = {} & [iky'\Psi_0 - \partial(\bar{\phi}_{s1}{}^{(as)**})/\partial x']\, \partial(\phi_1{}^{(s)*})/\partial x' - \\ & \{[ik(x'+a)+1]\Psi_0 - ikH_0 + \partial(\bar{\phi}_{s1}{}^{(as)**})/\partial y'\}\, \partial(\phi_1{}^{(s)*})/\partial y'\end{aligned} \tag{40}$$

in which $\phi_1{}^{(s)*}$ is defined by Eq. (26) and $\bar{\phi}_{s1}{}^{(as)**}$ is defined by

$$\bar{\phi}_{s1}{}^{(as)**} = [\bar{\phi}_1{}^{(as)}(x', y', +0) + \bar{\phi}_1{}^{(as)}(x', y', -0)]/2 \tag{41}$$

The pressure difference across the plane $y' = 0$ is given by

$$\overline{\Delta p}_{f2} + 2\rho U_0{}^2 \Phi_f = 2\rho U_0{}^2(ik + \partial/\partial x')\bar{\phi}_{f2}{}^{(as)*} \tag{42}$$

where $\bar{\phi}_{f2}{}^{(as)*}$ is defined by

$$\bar{\phi}_{f2}{}^{(as)*} = [\bar{\phi}_2{}^{(as)}(x', +0, z') - \bar{\phi}_2{}^{(as)}(x', -0, z')]/2 \tag{43}$$

and where

§ Hitch[2] pointed out that symmetric forces due to yawing of the tailplane at incidence is small and invariably ignored in the flutter calculation.

¶ The boundary-value problem for $\phi_2{}^{(s)}$ is derived by the vortex method in Ref. 9.

$$\Phi_f(x',z') = -(\partial\phi_1^{(s)}/\partial x')[\partial(\bar{\phi}_{f1}^{(as)*})/\partial x'] - (\partial\phi_1^{(s)}/\partial z')[\partial(\bar{\phi}_{f1}^{(as)*})/\partial z'] \quad (44)$$

in which $\bar{\phi}_{f1}^{(as)*}$ is already defined by Eq. (32). The tangency condition on the tailplane becomes

$$\bar{\phi}_{2z'}^{(as)} = ([-iky'\Psi_0 + \partial(\bar{\phi}_{s1}^{(as)**})/\partial x']\partial/\partial x' + \{[ik(x'+a)+1]\Psi_0 - ikH_0 + \partial(\bar{\phi}_{s1}^{(as)**})/\partial y'\}\,\partial/\partial y' - \partial^2(\bar{\phi}_{s1}^{(as)**})/\partial z'^2)h_{s0} \quad \text{on} \quad z'=0 \quad (45)$$

where $\partial^2(\bar{\phi}_{s1}^{(as)**})/\partial z'^2$ is defined by

$$\partial^2(\bar{\phi}_{s1}^{(as)**})/\partial z'^2 = \{[\partial^2\bar{\phi}_1^{(as)}/\partial z'^2]_{z'=+0} + [\partial^2\bar{\phi}_1^{(as)}/\partial z'^2]_{z'=-0}\}/2 \quad (46)$$

The tangency condition on the fin becomes

$$\bar{\phi}_{2y'}^{(as)} = (\Psi_0 + \partial\bar{h}_f/\partial x'\bar{q})\partial\phi_1^{(s)}/\partial x' + \partial\bar{h}_f/\partial z'\bar{q}\,\partial\phi_1^{(s)}/\partial z' - [\bar{h}_f\bar{q} + \Psi_0(x'+a) - H_0]\,\partial^2\phi_1^{(s)}/\partial y'^2 \quad \text{on} \quad y'=0 \quad (47)$$

In deriving Eqs. (45) and (47) the terms which do not produce pressure difference have been omitted.[9]

It is clear from the equations derived earlier that the prediction of the effect of yaw and sideslip of the tailplane is only possible by solving the second-order boundary-value problem, which requires the solutions of $\phi_1^{(s)}$ and $\phi_1^{(as)}$ to be known.

III. Integral Equations

The first-order boundary-value problems for $\phi_1^{(s)}$ and $\phi_1^{(as)}$ can be solved by applying the theories already developed.[5-8] We therefore concentrate on the solution of the second-order boundary-value problem by assuming that $\phi_1^{(s)}$ and $\phi_1^{(as)}$ are already known.

It is not difficult to derive the integral equations for $\bar{\phi}_2^{(as)}$ when we extend the approach employed in Ref. 10. The results are as follows:

$$\frac{\bar{W}_2^{(as)}}{U_0}(x',y^*) - b_0 l_s \int_{-1}^{1} d\eta^* \int_{\xi_{st}(\eta^*)}^{\infty} \left[\frac{\Phi_s^{(as)}(\xi',\eta^*)}{2\pi}\right] \times K_{ss}[k, x'-\xi', S_s(y^*-\eta^*)]\,d\xi' - b_0 l_f \int_0^1 d\zeta^* \int_{\xi_{ft}(\zeta^*)}^{\infty} \left[\frac{\Phi_f(\xi',\zeta^*)}{2\pi}\right] K_{sf}(k, x'-\xi', S_s y^*, S_f\zeta^*)\,d\xi' =$$
$$\frac{b_0 l_s}{4\pi\rho U_0^2}\int_{-1}^{1} d\eta^* \int_{\xi_{sl}(\eta^*)}^{\xi_{st}(\eta^*)} [\overline{\Delta p}_{s2}^{(as)}(\xi',\eta^*) + 2\rho U_0^2 \Phi_s^{(as)}(\xi',\eta^*)] \times K_{ss}[k, x'-\xi', S_s(y^*-\eta^*)]\,d\xi' +$$
$$\frac{b_0 l_f}{4\pi\rho U_0^2}\int_0^1 d\zeta^* \int_{\xi_{fl}(\zeta^*)}^{\xi_{ft}(\zeta^*)} [\overline{\Delta p}_{f2}(\xi',\zeta^*) + 2\rho U_0^2 \Phi_f(\xi',\zeta^*)] \times K_{sf}(k, x'-\xi', S_s y^*, S_f\zeta^*)\,d\xi' \quad (48)$$

$$\frac{\bar{V}_2}{U_0}(x',z^*) - b_0 l_s \int_{-1}^{1} d\eta^* \int_{\xi_{st}(\eta^*)}^{\infty} \left[\frac{\Phi_s^{(as)}(\xi',\eta^*)}{2\pi}\right] \times K_{fs}(k, x'-\xi', S_s\eta^*, S_f z^*)\,d\xi' - b_0 l_f \int_0^1 d\zeta^* \int_{\xi_{ft}(\zeta^*)}^{\infty} \left[\frac{\Phi_f(\xi',\zeta^*)}{2\pi}\right] K_{ff}[k, x'-\xi', S_f(z^*-\zeta^*)]\,d\xi' =$$
$$\frac{b_0 l_s}{4\pi\rho U_0^2}\int_{-1}^{1} d\eta^* \int_{\xi_{sl}(\eta^*)}^{\xi_{st}(\eta^*)} [\overline{\Delta p}_{s2}^{(as)}(\xi',\eta^*) + 2\rho U_0^2 \Phi_s^{(as)}(\xi',\eta^*)] K_{fs}(k, x'-\xi', S_s\eta^*, S_f z^*)\,d\xi' +$$
$$\frac{b_0 l_f}{4\pi\rho U_0^2}\int_0^1 d\zeta^* \int_{\xi_{fl}(\zeta^*)}^{\xi_{ft}(\zeta^*)} [\overline{\Delta p}_{f2}(\xi',\zeta^*) + 2\rho U_0^2 \Phi_f(\xi',\zeta^*)] \times K_{ff}[k, x'-\xi', S_f(z^*-\zeta^*)]\,d\xi' \quad (49)$$

where

$$\frac{\bar{W}_2^{(as)}}{U_0}(x',y^*) \equiv \left.\frac{\partial\bar{\phi}_2^{(as)}}{\partial z'}\right|_{z'=0} \quad (50)$$

which is given by Eq. (45), and

$$\frac{\bar{V}_2}{U_0}(x',z^*) \equiv \left.\frac{\partial\bar{\phi}_2^{(as)}}{\partial y'}\right|_{y'=0} \quad (51)$$

which is given by Eq. (47). K_{ss}, K_{sf}, K_{fs} and K_{ff} are the same kernel functions which appear in the first-order antisymmetric problem and hence equivalent to those derived by Davies.[6] They are given in the Appendix for completeness.

The first-order velocities like $\partial(\bar{\phi}_{s1}^{(as)**})/\partial x'$, $\partial(\phi_1^{(s)*})/\partial x', \ldots$ etc., which are necessary for the evaluation of $\Phi_s^{(as)}$, Φ_f, $\bar{W}_2^{(as)}/U_0$, and $\bar{V}_2/U_0$, are given as follows. From Eq. (25) and the boundary condition of $\Delta p_{s1}^{(s)} = 0$ off the tailplane, we obtain

$$\left.\begin{aligned} \frac{\partial\phi_1^{(s)*}}{\partial x'}(x',y') &= \frac{\Delta p_{s1}^{(s)}(x',y')}{2\rho U_0^2} && \text{for} \quad \xi_{sl} \leqq x' \leqq \xi_{st} \\ \frac{\partial\phi_1^{(s)*}}{\partial x'}(x',y') &= 0 && \text{for} \quad x' \geqq \xi_{st} \end{aligned}\right\} \quad (52)$$

$$\left.\begin{aligned} \frac{\partial\phi_1^{(s)*}}{\partial y'}(x',y') &= \frac{1}{2\rho U_0^2}\frac{\partial}{\partial y'}\int_{\xi_{sl}(y')}^{x'} \Delta p_{s1}^{(s)}(\lambda,y')\,d\lambda && \text{for} \quad \xi_{sl} \leqq x' \leqq \xi_{st} \\ \frac{\partial\phi_1^{(s)*}}{\partial y'}(x',y') &= \frac{1}{2\rho U_0^2}\frac{\partial}{\partial y'}\int_{\xi_{sl}(y')}^{\xi_{st}(y')} \Delta p_{s1}^{(s)}(\lambda,y')\,d\lambda && \text{for} \quad x' \geqq \xi_{st} \end{aligned}\right\} \quad (53)$$

From Eq. (30) and the boundary condition of $\overline{\Delta p}_{f1} = 0$ off the fin, we obtain

$$\left.\begin{aligned} \frac{\partial\bar{\phi}_{f1}^{(as)*}}{\partial x'}(x',z') &= -ike^{-ikx'}\int_{\xi_{fl}(z')}^{x'} e^{ik\lambda}\left[\frac{\overline{\Delta p}_{f1}(\lambda,z')}{2\rho U_0^2}\right]d\lambda + \frac{\overline{\Delta p}_{f1}(x',z')}{2\rho U_0^2} && \text{for} \quad \xi_{fl} \leqq x' \leqq \xi_{ft} \\ \frac{\partial\bar{\phi}_{f1}^{(as)*}}{\partial x'}(x',z') &= -ike^{-ikx'}\int_{\xi_{fl}(z')}^{\xi_{ft}(z')} e^{ik\lambda}\left[\frac{\overline{\Delta p}_{f1}(\lambda,z')}{2\rho U_0^2}\right]d\lambda && \text{for} \quad x' \geqq \xi_{ft} \end{aligned}\right\} \quad (54)$$

$$\left.\begin{aligned} \frac{\partial\bar{\phi}_{f1}^{(as)*}}{\partial z'}(x',z') &= e^{-ikx'}\frac{\partial}{\partial z'}\int_{\xi_{fl}(z')}^{x'} e^{ik\lambda}\left[\frac{\overline{\Delta p}_{f1}(\lambda,z')}{2\rho U_0^2}\right]d\lambda && \text{for} \quad \xi_{fl} \leqq x' \leqq \xi_{ft} \\ \frac{\partial\bar{\phi}_{f1}^{(as)*}}{\partial z'}(x',z') &= e^{-ikx'}\frac{\partial}{\partial z'}\int_{\xi_{fl}(z')}^{\xi_{ft}(z')} e^{ik\lambda}\left[\frac{\overline{\Delta p}_{f1}(\lambda,z')}{2\rho U_0^2}\right]d\lambda && \text{for} \quad x' \geqq \xi_{ft} \end{aligned}\right\} \quad (55)$$

Utilizing Green's formula, Eq. (30) and the boundary condition of $\overline{\Delta p}_{f1} = 0$ off the fin, we obtain

$$\frac{\partial\bar{\phi}_{s1}^{(as)**}}{\partial x'}(x',y') = \frac{b_0 l_f}{4\pi\rho U_0^2}\int_0^1 d\zeta^* \int_{\xi_{fl}(\zeta^*)}^{\xi_{ft}(\zeta^*)} \overline{\Delta p}_{f1}(\xi',\zeta^*) \times K_{sx}(k, x'-\xi', S_s y^*, S_f\zeta^*)\,d\xi' \quad (56)$$

$$\frac{\partial\bar{\phi}_{s1}^{(as)**}}{\partial y'}(x',y') = \frac{b_0 l_f}{4\pi\rho U_0^2}\int_0^1 d\zeta^* \int_{\xi_{fl}(\zeta^*)}^{\xi_{ft}(\zeta^*)} \overline{\Delta p}_{f1}(\xi',\zeta^*) \times K_{sy}(k, x'-\xi', S_s y^*, S_f\zeta^*)\,d\xi' \quad (57)$$

Utilizing Green's formula, Eq. (25) and the boundary condition of $\Delta p_{s1}^{(s)} = 0$ off the tailplane, we obtain

$$\frac{\partial\phi_1^{(s)}}{\partial x'}(x',0,z') = \frac{b_0 l_s}{4\pi\rho U_0^2}\int_{-1}^{1} d\eta^* \int_{\xi_{sl}(\eta^*)}^{\xi_{st}(\eta^*)} \Delta p_{s1}^{(s)}(\xi',\eta^*) \times K_{fx}(0, x'-\xi', S_s\eta^*, S_f z^*)\,d\xi' \quad (58)$$

$$\frac{\partial\phi_1^{(s)}}{\partial z'}(x',0,z') = \frac{b_0 l_s}{4\pi\rho U_0^2}\int_{-1}^{1} d\eta^* \int_{\xi_{sl}(\eta^*)}^{\xi_{st}(\eta^*)} \Delta p_{s1}^{(s)}(\xi',\eta^*) \times K_{fz}(0, x'-\xi', S_s\eta^*, S_f z^*)\,d\xi' \quad (59)$$

The kernel functions which appear in Eqs. (56–59) can be derived by the approach analogous to that used in Ref. 13, and they are given in the Appendix. These kernel functions are nonsingular as far as the point (x',y') or (x',z') are not located at the junction of the tailplane and fin.

As to the derivative, $\partial^2(\bar{\phi}_{s1}^{(as)**})/\partial z'^2$, of the first-order induced velocity, which appears in the tangency condition on the tailplane [Eq. (45)], a similar expression could be derived as

$$\frac{\partial^2 \bar{\phi}_{s1}{}^{(as)**}}{\partial z'^2}(x', y') = \frac{b_0 l_f}{4\pi\rho U_0{}^2}\int_0^1 d\zeta^* \int_{\xi_{fl}(\zeta^*)}^{\xi_{ft}(\zeta^*)} \overline{\Delta p}_{f1}(\xi', \zeta^*) \times K_{szz}(k, x'-\xi', S_s y^*, S_f \zeta^*)\, d\xi' \tag{60}$$

where

$$K_{szz} = -\frac{e^{-ik(x'-\xi')}}{b_0{}^2}\lim_{\substack{\eta'\to 0\\ z'\to 0}}\left(\frac{\partial^2}{\partial z'^2\,\partial y'}\int_{-\infty}^{x'-\xi'} e^{ik\lambda^*} \times \left\{\frac{1}{[\lambda^{*2}+(y'-\eta')^2+(z'-\zeta')^2]^{1/2}}\right\} d\lambda^*\right) \tag{61}$$

It was found, however, that the derivative shows a strong singular behavior for the control points near the junction (it becomes infinite at the junction). It should be noticed that the derivative comes from Taylor series expansion of $\phi_{z'}(x', y', h_s \pm 0, t)$ about $z' = \pm 0$, which is prohibited at the junction, in the process of transfer of the tangency condition. Therefore, Eq. (60) is questionable for the control points (about $y^* < 0.3$) near the junction, for which a different approach (it is not found yet) is necessary. Fortunately, however, the values of the derivative, $\partial^2(\bar{\phi}_{s1}{}^{(as)**})/\partial z'^2$, for about $y^* > 0.3$ is so small that the term $\partial^2(\bar{\phi}_{s1}{}^{(as)**})/dz'^2 h_{s0}$ in Eq. (45) can be neglected in comparison with the other terms. This suggests that the contribution of the term, $\partial^2(\bar{\phi}_{s1}{}^{(as)**})/\partial z'^2 h_{s0}$ in Eq. (45), to the rolling moment of the tailplane, which is important for a T-tail flutter, is very small. It seems, therefore, reasonable to drop the term $\partial^2(\bar{\phi}_{s1}{}^{(as)**})/\partial z'^2 h_{s0}$ in Eq. (45) completely in the subsequent analysis. For similar reasons we will also drop the term, $\partial^2\phi_1{}^{(s)}/\partial y'^2[\bar{h}_f \bar{q} + \Psi_0(x'+a) - H_0]$ in Eq. (47), in the subsequent analysis.

IV. Method for Numerical Solution

The details of the method for solving the integral equation for the second-order antisymmetric load distributions, $\overline{\Delta p}_{s2}{}^{(as)}$ and $\overline{\Delta p}_{f2}$, are given in this section.

1. Assumed Loading Functions

By the same reason discussed in Ref. 10, we let the unknown functions of Eqs. (48) and (49) be

$$\overline{\Delta p}_{s2}{}^{(as)*} = \overline{\Delta p}_{s2}{}^{(as)} + 2\rho U_0{}^2 \Phi_s{}^{(as)} \tag{62}$$

and

$$\overline{\Delta p}_{f2}{}^{*} = \overline{\Delta p}_{f2} + 2\rho U_0{}^2 \Phi_f \tag{63}$$

rather than $\overline{\Delta p}_{s2}{}^{(as)}$ and $\overline{\Delta p}_{f2}$ themselves. The loading function for $\overline{\Delta p}_{s2}{}^{(as)*}$ is assumed as

$$\overline{\Delta p}_{s2}{}^{(as)*} = 4\pi\rho U_0{}^2 (l_s/b_0) L_{s2}{}^{(as)}(\theta, \eta^*)\bar{q} \tag{64}$$

where

$$L_{s2}{}^{(as)}(\theta, \eta^*) = (b_0/b_s)\sum_{n=0} l_n{}^{(2)}(\theta) S_n{}^{(as)(2)}(\eta^*) \tag{65}$$

where

$$\left.\begin{aligned} l_0{}^{(2)}(\theta) &= 1 - [1-(1-\cos\theta)^2/4]^{1/2} && n = 0\\ l_1{}^{(2)}(\theta) &= \cot\frac{\theta}{2} && n = 1\\ l_n{}^{(2)}(\theta) &= \frac{4}{2^{2(n-1)}}\sin(n-1)\theta && n \geqq 2 \end{aligned}\right\} \tag{66}$$

$$S_n{}^{(as)(2)}(\eta^*) = (1-\eta^{*2})^{1/2}\sum_{m=0} (|\eta^*|/\eta^*)^{m+1} a_{s,nm}{}^{(as)(2)} \eta^{*m} \tag{67}$$

where the coefficients $a_{s,nm}{}^{(as)(2)}$ are the unknown factors to be determined. In Eq. (64), the angular variable θ is defined by

$$\left.\begin{aligned} \xi' &= \xi_{sm} - (b_s/b_0)\cos\theta\\ \xi_{sm} &= (\xi_{st} + \xi_{sl})/2\\ b_s/b_0 &= (\xi_{st} - \xi_{sl})/2 \end{aligned}\right\} \quad \text{for} \quad 0 \leqq \theta \leqq \pi \tag{68}$$

The loading function for $\overline{\Delta p}_{f2}{}^{*}$ is assumed as

$$\overline{\Delta p}_{f2}{}^{*} = 4\pi\rho U_0{}^2 (l_f/b_0) L_{f2}(\theta, \zeta^*)\bar{q} \tag{69}$$

where

$$L_{f2}(\theta, \zeta^*) = (b_0/b_f)\sum_{n=0} l_n{}^{(2)}(\theta) F_n{}^{(2)}(\zeta^*) \tag{70}$$

in which

$$F_n{}^{(2)}(\zeta^*) = (1-\zeta^*)^{1/2}\sum_{m=0} \zeta^{*m} a_{f,nm}{}^{(2)} \tag{71}$$

where the coefficients $a_{f,nm}{}^{(2)}$ are the unknown factors to be determined. In Eq. (69), the angular variable θ is defined by

$$\left.\begin{aligned} \xi' &= \xi_{fm} - (b_f/b_0)\cos\theta\\ \xi_{fm} &= (\xi_{ft} + \xi_{fl})/2\\ b_f/b_0 &= (\xi_{ft} - \xi_{fl})/2 \end{aligned}\right\} \quad \text{for} \quad 0 \leqq \theta \leqq \pi \tag{72}$$

The loading function $\{1-[1-(1-\cos\theta)^2/4]^{1/2}\}$ in Eqs. (64) and (69) are introduced in order to meet the logarithmically infinite downwash at the trailing edges.[10] It should be noticed that $\overline{\Delta p}_{s2}{}^{(as)*}$ and $\overline{\Delta p}_{f2}{}^{*}$ do not become zero but remain finite at the trailing edges.[10] The values of $\overline{\Delta p}_{s2}{}^{(as)*}$ and $\overline{\Delta p}_{f2}{}^{*}$ at the trailing edges are determined only by solving the integral equations and it is of interest that the values thus obtained are such that the true unknown loads $\overline{\Delta p}_{s2}{}^{(as)}$ and $\overline{\Delta p}_{f2}$ calculated by the use of Eqs. (62) and (63) automatically satisfy the Kutta condition.

2. Working Form of the Integral Equations

We define modified forms of the kernel functions $\bar{K}_{ss}$ and $\bar{K}_{ff}$ by

$$K_{ss}[k, x'-\xi', S_s(y^*-\eta^*)] = \bar{K}_{ss}[k, x'-\xi', S_s(y^*-\eta^*)]/[b_0{}^2 S_s{}^2(y^*-\eta^*)^2] \tag{73}$$

$$K_{ff}[k, x'-\xi', S_f(z^*-\zeta^*)] = \bar{K}_{ff}[k, x'-\xi', S_f(z^*-\zeta^*)]/[b_0{}^2 S_f{}^2(z^*-\zeta^*)^2] \tag{74}$$

Substituting Eqs. (64–74) into Eqs. (48) and (49), we obtain the following working form of the integral equations:

$$\begin{aligned} &\frac{\bar{W}_2{}^{(as)}}{U_0}(x', y^*) - \frac{1}{S_s}\oint_{-1}^{1}\frac{d\eta^*}{(y^*-\eta^*)^2}\int_{\xi_{st}(\eta^*)}^{\infty}\left[\frac{\Phi_s{}^{(as)}(\xi', \eta^*)}{2\pi}\right] \times \bar{K}_{ss}[k, x'-\xi', S_s(y^*-\eta^*)]\, d\xi' -\\ &S_f\int_0^1 d\zeta^*\int_{\xi_{ft}(\zeta^*)}^{\infty}\left[\frac{\Phi_f(\xi', \zeta^*)}{2\pi}\right] \times \bar{K}_{sf}(k, x'-\xi', S_s y^*, S_f\zeta^*)\, d\xi' =\\ &\bar{q}\sum_{n=0}\sum_{m=0} a_{s,nm}{}^{(as)(2)}\oint_{-1}^{1}\frac{(|\eta^*|/\eta^*)^{m+1}\eta^{*m}(1-\eta^{*2})^{1/2}}{(y^*-\eta^*)^2} \times d\eta^*\int_0^\pi l_n{}^{(2)}(\theta)\bar{K}_{ss}[k, \theta, S_s(y^*-\eta^*)] \times\\ &\sin\theta\, d\theta + \bar{q}S_f{}^2\sum_{n=0}\sum_{m=0} a_{f,nm}{}^{(2)}\int_0^1 (1-\zeta^*)^{1/2}\zeta^{*m} \times d\zeta^*\int_0^\pi l_n{}^{(2)}(\theta)\bar{K}_{sf}(k, \theta, S_s y^*, S_f\zeta^*)\sin\theta\, d\theta \end{aligned} \tag{75}$$

$$\begin{aligned} &\frac{\bar{V}_2}{U_0}(x', z^*) - S_s\int_{-1}^{1} d\eta^*\int_{\xi_{st}(\eta^*)}^{\infty}\left[\frac{\Phi_s{}^{(as)}(\xi', \eta^*)}{2\pi}\right] \times \bar{K}_{fs}(k, x'-\xi', S_s\eta^*, S_f z^*)\, d\xi' -\\ &\frac{1}{S_f}\oint_0^1\frac{d\zeta^*}{(z^*-\zeta^*)^2}\int_{\xi_{ft}(\zeta^*)}^{\infty}\left[\frac{\Phi_f(\xi', \zeta^*)}{2\pi}\right] \times \bar{K}_{ff}[k, x'-\xi', S_f(z^*-\zeta^*)]\, d\xi' =\\ &\bar{q}S_s{}^2\sum_{n=0}\sum_{m=0} a_{s,nm}{}^{(as)(2)}\int_{-1}^{1}(|\eta^*|/\eta^*)^{m+1}\eta^{*m}(1-\eta^{*2})^{1/2} \times d\eta^*\int_0^\pi l_n{}^{(2)}(\theta)\bar{K}_{fs}(k, \theta, S_s\eta^*, S_f z^*) \times\\ &\sin\theta\, d\theta + \bar{q}\sum_{n=0}\sum_{m=0} a_{f,nm}{}^{(2)}\oint_0^1\frac{(1-\zeta^*)^{1/2}\zeta^{*m}}{(z^*-\zeta^*)^2} \times d\zeta^*\int_0^\pi l_n{}^{(2)}(\theta)\bar{K}_{ff}[k, \theta, S_f(z^*-\zeta^*)]\sin\theta\, d\theta \end{aligned} \tag{76}$$

In Eqs. (75) and (76), $\bar{K}_{sf}$ and $\bar{K}_{fs}$ are defined by

$$K_{sf} = \bar{K}_{sf}/b_0{}^2, \qquad K_{fs} = \bar{K}_{fs}/b_0{}^2 \tag{77}$$

As to the evaluation of the definite integrals in Eqs. (75) and

(76), the method which is analogous to that developed by Watkins et al.[5] for a usual lifting surface is applied. All the chordwise integrals on the surfaces are evaluated by applying two 10-point gaussian integration formula. For example, the chordwise integral in Eq. (75) can be rewritten as

$$f_n(\eta^*) = \int_0^{\theta_x} l_n^{(2)}(\theta)\bar{K}_{ss}[k, \theta, S_s(y^* - \eta^*)] \sin\theta \, d\theta + \int_{\theta_x}^{\pi} l_n^{(2)}(\theta)\bar{K}_{ss}[k, \theta, S_s(y^* - \eta^*)] \sin\theta \, d\theta$$

where θ_x denotes the value of θ which corresponds to $x' - \xi' = 0$. To each integration region, 10-point gaussian integration formula is applied. The finite discontinuity which appears in the variation chordwise of $\bar{K}_{ss}[k, \theta, S_s(y^* - \eta^*)]$ when $y^* = \eta^*$ is, therefore, properly taken into account.[5] The chordwise integrals in the wake regions are evaluated by applying Simpson's formula. 20-point formula for the region from the trailing edge to one chord downstream and 16-point formula for that from one chord downstream to 5-chord downstream are employed. No appreciable changes in the results are observed by extending the integration region further downstream. As to the treatment of the spanwise improper integrals, the same procedure developed by Watkins et al.[5] is employed.

3. Working Forms of the First-Order Velocities

In order to evaluate the normal wash conditions $\bar{W}_2^{(as)}/U_0$ and $\bar{V}_2/U_0$, and $\Phi_s^{(as)}$ and Φ_f, the first-order velocities must be calculated. Let the first-order solutions, $\Delta p_{s1}^{(s)}$ and $\overline{\Delta p}_{f1}$, be given by

$$\Delta p_{s1}^{(s)}(\theta, \eta^*) = 4\pi\rho U_0^2(l_s/b_0)\alpha(b_0/b_s) \sum_{n=0} l_n^{(1)}(\theta) S_n^{(s)(1)}(\eta^*) \tag{78}$$

$$\overline{\Delta p}_{f1}(\theta, \zeta^*) = 4\pi\rho U_0^2(l_f/b_0)\bar{q}(b_0/b_f) \sum_{n=0} l_n^{(1)}(\theta) F_n^{(1)}(\zeta^*) \tag{79}$$

where

$$\left.\begin{aligned} l_0^{(1)}(\theta) &= \cot\frac{\theta}{2} & n &= 0 \\ l_n^{(1)}(\theta) &= \frac{4}{2^{2n}}\sin n\theta & n &\geqq 1 \end{aligned}\right\} \tag{80}$$

and where

$$S_n^{(s)(1)}(\eta^*) = (1 - \eta^{*2})^{1/2} \sum_{m=0} \eta^{*2m} a_{s,nm}^{(s)(1)} \tag{81}$$

and

$$F_n^{(1)}(\zeta^*) = (1 - \zeta^*)^{1/2} \sum_{m=0} \zeta^{*m} a_{f,nm}^{(1)} \tag{82}$$

where the coefficients $a_{s,nm}^{(s)(1)}$ and $a_{f,nm}^{(1)}$ are factors which have been determined by solving the first-order integral equations (see the next section). Then the working forms of the first-order velocities are given by substituting Eqs. (81) and (82) into Eqs. (52–59) as follows:

$$\left.\begin{aligned} \frac{\partial\phi_1^{(s)*}}{\partial x'}(\theta, y^*) &= 2\pi\alpha\left(\frac{l_s}{b_0}\right)\left(\frac{b_0}{b_s}\right)\sum_{n=0} l_n^{(1)}(\theta) S_n^{(s)(1)}(y^*) & &\text{for } 0 \leqq \theta \leqq \pi \\ \frac{\partial\phi_1^{(s)*}}{\partial x'}(x', y^*) &= 0 & &\text{for } x' \geqq \xi_{st} \end{aligned}\right\} \tag{83}$$

$$\frac{\partial\phi_1^{(s)*}}{\partial y'}(\theta, y^*) = 2\pi\alpha\left\{\sum_{n=0} X_n(\theta)[Y_n^{\mathrm{I}}(y^*) + Y_n^{\mathrm{II}}(y^*)] + \left(\frac{b_0}{b_s}\right)\sum_{n=0} l_n^{(1)}(\theta) S_n^{(s)(1)}(y^*)\left[\frac{d(b_s/b_0)}{dy^*}\cos\theta - \frac{d\xi_{sm}}{dy^*}\right]\right\} \quad \text{for } 0 \leqq \theta \leqq \pi \tag{84}$$

$$\frac{\partial\phi_1^{(s)*}}{\partial y'}(x', y^*) = 2\pi\alpha\left\{\sum_{n=0} X_n(\pi)[Y_n^{\mathrm{I}}(y^*) + Y_n^{\mathrm{II}}(y^*)]\right\} \quad \text{for } x' \geqq \xi_{st} \tag{85}$$

where $X_n(\theta)$ are given by integrating $l_n^{(1)}(\theta)\sin\theta$ with respect to θ as

$$\begin{aligned} X_0(\theta) &= \theta + \sin\theta & n &= 0 \\ X_1(\theta) &= \frac{\theta}{2} - \frac{\sin 2\theta}{4} & n &= 1 \\ X_n(\theta) &= \frac{4}{2^{2n}}\left[\frac{\sin(n-1)\theta}{2(n-1)} - \frac{\sin(n+1)\theta}{2(n+1)}\right] & n &\geqq 2 \end{aligned} \tag{86}$$

and where $Y_n^{\mathrm{I}}(y^*)$ and $Y_n^{\mathrm{II}}(y^*)$ are given by differentiating $S_n^{(s)(1)}(y^*)$ by y^* as

$$\left.\begin{aligned} Y_n^{\mathrm{I}}(y^*) &= -y^*/(1 - y^{*2})^{1/2} \sum_{m=0} a_{s,nm}^{(s)(1)} y^{*2m} \\ Y_n^{\mathrm{II}}(y^*) &= 2y^*(1 - y^{*2})^{1/2} \sum_{m=1} m a_{s,nm}^{(s)(1)} y^{*2(m-1)} \end{aligned}\right\} \tag{87}$$

$$\frac{\partial\bar{\phi}_{f1}^{(as)*}}{\partial x'}(\theta, z^*) = 2\pi\left\{-ik\exp\left[-ik\left(\xi_{fm} - \frac{b_f}{b_0}\cos\theta\right)\right] \times \int_0^{\theta}\exp\left[ik\left(\xi_{fm} - \frac{b_f}{b_0}\cos\theta'\right)\right]\sum_{n=0} l_n^{(1)}(\theta')F_n^{(1)}(z^*) \times \sin\theta'\,d\theta' + S_f(b_0/b_f)\sum_{n=0} l_n^{(1)}(\theta)F_n^{(1)}(z^*)\right\}\bar{q} \quad \text{for } 0 \leqq \theta \leqq \pi \tag{88}$$

$$\frac{\partial\bar{\phi}_{f1}^{(as)*}}{\partial x'}(x', z^*) = 2\pi\left\{-ik\exp(-ikx')\int_0^{\pi} \times \exp\left[ik\left(\xi_{fm} - \frac{b_f}{b_0}\cos\theta'\right)\right]\sum_{n=0} l_n^{(1)}(\theta')F_n^{(1)}(z^*)\sin\theta'\,d\theta'\right\}\bar{q} \quad \text{for } x' \geqq \xi_{ft} \tag{89}$$

$$\frac{\partial\bar{\phi}_{f1}^{(as)*}}{\partial z'}(\theta, z^*) = 2\pi\left\{\left[\frac{d(b_f/b_0)}{dz^*}\cos\theta - \frac{d\xi_{fm}}{dz^*}\right]\left(\frac{b_0}{b_f}\right) \times \left[\sum_{n=0} l_n^{(1)}(\theta)F_n^{(1)}(z^*)\right] + \exp\left[-ik\left(\xi_{fm} - \frac{b_f}{b_0}\cos\theta\right)\right] \times \int_0^{\theta}\exp\left[ik\left(\xi_{fm} - \frac{b_f}{b_0}\cos\theta'\right)\right]\left[\sum_{n=0} l_n^{(1)}(\theta')\frac{dF_n^{(1)}(z^*)}{dz^*}\right] \times \sin\theta'\,d\theta'\right\}\bar{q} \quad \text{for } 0 \leqq \theta \leqq \pi \tag{90}$$

$$\frac{\partial\bar{\phi}_{f1}^{(as)*}}{\partial z'}(x', z^*) = 2\pi\exp(-ikx') \times \left\{\int_0^{\pi}\exp\left[ik\left(\xi_{fm} - \frac{b_f}{b_0}\cos\theta'\right)\right] \times \left[\sum_{n=0} l_n^{(1)}(\theta')\frac{dF_n^{(1)}(z^*)}{dz^*}\right]\sin\theta'\,d\theta'\right\}\bar{q} \quad \text{for } x' \geqq \xi_{ft} \tag{91}$$

The working forms of the induced velocities, $\partial(\bar{\Phi}_{s1}^{(as)**})/\partial x'$, $\partial(\bar{\Phi}_{s1}^{(as)**})/\partial y'$, $\partial\phi_1^{(s)}/\partial x'$ and $\partial\phi_1^{(s)}/\partial z'$, are not given here since they can be easily derived from Eqs. (56–59), respectively, by the use of Eqs. (78) and (79). Ten-point gaussian integration formula is applied for evaluating the chordwise integrals in Eqs. (88–91). As to the surface integrals in Eqs. (56–59), 10-point gaussian integration formula is also employed for evaluating both the chordwise and spanwise definite integrals.

In evaluating the normal wash conditions on the surfaces, we have only to calculate the induced velocities at the limited number of control points (72 control points were employed for the numerical examples given in Sec. VI). On the other hand, we need to evaluate the induced velocities, which appear in $\Phi_s^{(as)}$ and Φ_f in the wake integrals, at every integration point. This may consume an almost prohibitive amount of computing time if we perform the surface integrals of Eqs. (57–59) directly at each integration point (about 85,000 integration points for the numerical examples in Sec. VI). It is, therefore, desirable to use simpler expressions which approximate the true distributions of the induced velocities in the wake. This can be accomplished by approximating the distribution of the induced velocities by the polinomial series whose unknown coefficients are determined by collocation at the limited number of collocation points (36 points for the numerical examples in Sec. VI).

It is to be mentioned that the induced velocity, $\partial(\bar{\Phi}_{s1}^{(as)**})/\partial y'$, which appears in the normal wash condition, Eq. (45), becomes infinite as y' approaches zero. This singular behavior of $\partial(\bar{\phi}_{s1}^{(as)**})/\partial y'$ near the junction, which is physically inadmissible, becomes appreciable, however, only for the points located within about 7.5% of semispan from the junction. In the present procedure, therefore, the singularity is ignored by locating the nearest control points to the junction remote 11% of semispan from the junction. In evaluating $\Phi_s^{(as)}(x', y')$ [Eq. (40)] in which the term $\partial(\bar{\phi}_{s1}^{(as)**})/\partial y'$ also appears in the form of the product $\partial(\bar{\phi}_{s1}^{(as)**})/\partial y' \cdot \partial(\phi_1^{(s)*})/\partial y'$, no account has been taken of the singularity at the junction because it does not give appreciable effect on the rolling moment acting on the tailplane.

By equating both sides of Eqs. (75) and (76) at the limited number of control points, a set of linear simultaneous algebraic equations for the unknown coefficients $a_{s,nm}^{(as)(2)}$ and $a_{f,nm}^{(2)}$ are obtained. The true load distributions $\overline{\Delta p}_{s2}^{(as)}$ and $\overline{\Delta p}_{f2}$ will be obtained by the use of Eqs. (62) and (63).

V. Remarks on the Method for Solving the First-Order Integral Equations

1. First-Order Symmetric Load Distribution $\Delta p_{s1}^{(s)}$

The integral equation for $\Delta p_{s1}^{(s)}$ is solved by the direct application of the procedure developed by Watkins et al.[5]

2. First-Order Antisymmetric Load Distributions $\overline{\Delta p}_{s1}^{(as)}$ and $\overline{\Delta p}_{f1}$

Although methods for solving the first-order antisymmetric integral equations have already been proposed by several authors,[6-8] our own method which is similar to that employed in solving the second-order integral equations is used in the present report.

The integral equations for $\overline{\Delta p}_{s1}^{(as)}$ and $\overline{\Delta p}_{f1}$ are derived from the boundary-value problem for $\bar{\phi}_1^{(as)}$ [Eqs. (28–34)] as

$$\frac{\bar{W}_1^{(as)}}{U_0}(x', y^*) = \frac{b_0 l_s}{4\pi\rho U_0^2}\int_{-1}^{1} d\eta^* \int_{\xi_{sl}(\eta^*)}^{\xi_{st}(\eta^*)} \overline{\Delta p}_{s1}^{(as)}(\xi', \eta^*) \times K_{ss}[k, x'-\xi', S_s(y^*-\eta^*)]\, d\xi' + \frac{b_0 l_f}{4\pi\rho U_0^2}\int_0^1 d\zeta^* \int_{\xi_{fl}(\zeta^*)}^{\xi_{ft}(\zeta^*)} \overline{\Delta p}_{f1}(\xi', \zeta^*) \times K_{sf}(k, x'-\xi', S_s y^*, S_f \zeta^*)\, d\xi' \tag{92}$$

$$\frac{\bar{V}_1}{U_0}(x', z^*) = \frac{b_0 l_s}{4\pi\rho U_0^2}\int_{-1}^{1} d\eta^* \int_{\xi_{sl}(\eta^*)}^{\xi_{st}(\eta^*)} \overline{\Delta p}_{s1}^{(as)}(\xi', \eta^*) \times K_{fs}(k, x'-\xi', S_s\eta^*, S_f z^*)\, d\xi' + \frac{b_0 l_f}{4\pi\rho U_0^2}\int_0^1 d\zeta^* \int_{\xi_{fl}(\zeta^*)}^{\xi_{ft}(\zeta^*)} \overline{\Delta p}_{f1}(\xi', \zeta^*) \times K_{ff}[k, x'-\xi', S_f(z^*-\zeta^*)]\, d\xi' \tag{93}$$

where the kernel functions K_{ss}, K_{sf}, K_{fs} and K_{ff} are the same as those which have already appeared in the second-order integral equations, and where

$$\frac{\bar{W}_1^{(as)}}{U_0}(x', y^*) \equiv \bar{\phi}_{1z'}^{(as)}(x', y^*, 0)$$
$$\frac{\bar{V}_1}{U_0}(x', z^*) \equiv \bar{\phi}_{1y'}^{(as)}(x', 0, z^*) \tag{94}$$

which are given by Eqs. (33) and (34), respectively. We assume the unknown load distributions as

$$\overline{\Delta p}_{s1}^{(as)}(\theta, \eta^*) = 4\pi\rho U_0^2\left(\frac{l_s}{b_0}\right)\bar{q}\left(\frac{b_0}{b_s}\right)\sum_{n=0} l_n^{(1)}(\theta) S_n^{(as)(1)}(\eta^*) \tag{95}$$

where

$$S_n^{(as)(1)}(\eta^*) = (1-\eta^{*2})^{1/2}\sum_{m=0}(|\eta^*|/\eta^*)^{m+1}\eta^{*m} a_{s,nm}^{(as)(1)} \tag{96}$$

and where $l_n^{(1)}(\theta)$ is already given by Eq. (80) and $\overline{\Delta p}_{f1}(\theta, \zeta^*)$ is assumed to be given by Eq. (79). The coefficients $a_{s,nm}^{(as)(1)}$ in Eqs. (95) and $a_{f,nm}^{(1)}$ in Eq. (82) are the unknown factors to be determined.

Substituting Eqs. (95) and (79) into Eqs. (92) and (93), we obtain a working form of the integral equations as follows:

$$\frac{\bar{W}_1^{(as)}}{U_0}(x', y^*) = \bar{q}\sum_{n=0}\sum_{m=0} a_{s,nm}^{(as)(1)} ⨍_{-1}^{1} \frac{(|\eta^*|/\eta^*)^{m+1}\eta^{*m}(1-\eta^{*2})^{1/2}}{(y^*-\eta^*)^2} \times d\eta^* \int_0^\pi l_n^{(1)}(\theta)\bar{K}_{ss}[k, \theta, S_s(y^*-\eta^*)] \sin\theta\, d\theta + qS_f^2\sum_{n=0}\sum_{m=0} a_{f,nm}^{(1)}\int_0^1 (1-\zeta^*)^{1/2}\zeta^{*m} d\zeta^* \int_0^\pi l_n^{(1)}(\theta) \times \bar{K}_{sf}(k, \theta, S_s y^*, S_f\zeta^*)\sin\theta\, d\theta \tag{97}$$

$$\frac{\bar{V}_1}{U_0}(x', z^*) = \bar{q}S_s^2\sum_{n=0}\sum_{m=0} a_{s,nm}^{(as)(1)}\int_{-1}^{1} (|\eta^*|/\eta^*)^{m+1}\eta^{*m}(1-\eta^{*2})^{1/2} \times d\eta^* \int_0^\pi l_n^{(1)}(\theta)\bar{K}_{fs}(k, \theta, S_s\eta^*, S_f z^*)\sin\theta\, d\theta + \bar{q}\sum_{n=0}\sum_{m=0} a_{f,nm}^{(1)} ⨍_0^1 \frac{(1-\zeta^*)^{1/2}\zeta^{*m}}{(z^*-\zeta^*)^2} d\zeta^* \int_0^\pi l_n^{(1)}(\theta) \times \bar{K}_{ff}[k, \theta, S_f(z^*-\zeta^*)]\sin\theta\, d\theta \tag{98}$$

For the evaluation of the chordwise and spanwise definite integrals and for the treatment of the spanwise improper integrals, exactly the same procedure as in Sec. IV is applied.

In order to check the validity of the present procedure, Davies' generalized aerodynamic forces were calculated for a simplified T-tail (see p. 43 of Ref. 6). The tailplane is rectangular and of aspect ratio 1 and the fin is rectangular and of aspect ratio 1. In our notations, Davies' generalized aerodynamic forces are defined as follows:

$$Q_{p,q} = \frac{P_{p,q}}{8b_0^3\rho U_0^2} \tag{99}$$

where

$$P_{p,q} = b_0^3 S_s\int_{-1}^{1} d\eta^* \int_{\xi_{sl}}^{\xi_{st}} \bar{h}_{s,p}(\xi', S_s\eta^*)\overline{\Delta p}_{s,q}^{(as)(1)}(\xi', S_s\eta^*)\, d\xi' + b_0^3 S_f\int_0^1 d\zeta^* \int_{\xi_{fl}}^{\xi_{ft}} \bar{h}_{f,p}(\xi', S_f\zeta^*)\overline{\Delta p}_{f,q}^{(1)}(\xi', S_f\zeta^*)\, d\xi' \tag{100}$$

where $\bar{h}_{s,p}$ and $\bar{h}_{f,p}$ is the pth mode of the tailplane and of the fin respectively and where $\overline{\Delta p}_{s,q}^{(as)(1)}$ and $\overline{\Delta p}_{f,q}^{(1)}$ are the load distributions on the tailplane and on the fin, respectively, due to the qth mode of oscillation of the surfaces.

In the present example, the T-tail is assumed to be oscillating in one of four modes of oscillation defined[6] by

$$\left.\begin{array}{ll} \bar{h}_{s,1}(x', S_s y^*) = 0 & \bar{h}_{f,1}(x', S_f z^*) = 2 \\ \bar{h}_{s,2}(x', S_s y^*) = 0 & \bar{h}_{f,2}(x', S_f z^*) = x' \\ \bar{h}_{s,3}(x', S_s y^*) = S_s y^* & \bar{h}_{f,3}(x', S_f z^*) = S_f(3/2 - z^*) \\ \bar{h}_{s,4}(x', S_s y^*) = S_s y^* & \bar{h}_{f,4}(x', S_f z^*) = 0 \end{array}\right\} \tag{101}$$

A converged solution was obtained by using 24 control points on the tailplane (3 chordwise and 8 spanwise) and 24 control points on the fin (3 chordwise and 8 spanwise). The chordwise locations of the control points on the tailplane were $x' = -0.5$, $x' = 0$ and $x' = 0.5$. The spanwise locations were 11%, 22%, 33%, 44%, 55%, 66%, 77% and 88% semispan points from the midspan. The same chordwise and spanwise locations of the control points as those used for the tailplane were also employed for the fin. The results are compared with those calculated by Davies in Table 1. The agreement of the two methods is satisfactory.

VI. Examples

In order to show the applicability of the present theory, some numerical examples on simplified T-tails oscillating in yaw for which the experimental data are available[11] are given. As a first example we consider a T-tail whose tailplane is rectangular and of aspect ratio 3.0 and whose fin is rectangular and of

Table 1 Comparison of generalized aerodynamic forces (first-order solution) on simplified T-tail oscillating in yaw

Generalized aerodynamic forces	$k=0$ Davies	$k=0$ Present method	$k=0.25$ Davies	$k=0.25$ Present method	$k=0.5$ Davies	$k=0.5$ Present method
$Q_{12}'+iQ_{12}''$	−1.0865	−1.0744	−1.0748 −i0.3972	−1.0623 −i0.3983	−1.0640 −i0.8116	−1.0480 −i0.8116
$Q_{22}'+iQ_{22}''$	0.3282	0.3235	0.3300 −i0.0490	0.3254 −i0.0477	0.3418 −i0.0936	0.3376 −i0.0908
$Q_{32}'+iQ_{32}''$	−1.2306	−1.2094	−1.2172 −i0.4358	−1.1957 −i0.4395	−1.2047 −i0.8913	−1.1792 −i0.8962
$Q_{42}'+iQ_{42}''$	−0.0717	−0.0681	−0.0708 −i0.0187	−0.0672 −i0.0199	−0.0698 −i0.0386	−0.0661 −i0.0410

aspect ratio 3.0. The axis of yaw is through the midchord of the fin. The angle of attack of the tailplane is 5°29′. As the model used in the experiment[11] had a reflector plate at the base of the fin (the aspect ratio of the fin was 1.5), its effect has been taken into account in the theoretical model by doubling the height of the fin.[12]

h_{s0}, $\bar{h}_s$ and $\bar{h}_f$ can be expressed as

$$h_{s0} = \alpha x' \tag{102}$$

$$\bar{h}_s = 0 \tag{103}$$

$$\bar{h}_f = -x' \tag{104}$$

The generalized coordinate can be taken as $\bar{q} = \Psi_0$.

1. First-Order Antisymmetric Load Distributions $\overline{\Delta p}_{s1}^{(as)}$ and $\overline{\Delta p}_{f1}$

Substituting Eqs. (103) and (104) into Eqs. (33) and (34) respectively, the normal wash conditions on the surfaces become

$$\bar{W}_1^{(as)}(x', y^*)/(U_0\Psi_0) = 0$$
$$\bar{V}_1(x', z^*)/(U_0\Psi_0) = -(ikx'+1) \tag{105}$$

With these normal wash conditions, Eqs. (97) and (98) are solved for the unknown coefficients $a_{s,nm}^{(as)(1)}$ and $a_{f,nm}^{(1)}$. The same number of control points and the same locations of them as those described in Sec. V are used. The first-order rolling moments** acting on the tailplane calculated by integrating $\overline{\Delta p}_{s1}^{(as)}$ are plotted by the dotted line against the reduced frequency in Fig. 5. They are compared with the experimental values[11] obtained at $\alpha \doteqdot 0°$ since the rolling moments acting on the tailplane are only the first-order ones at $\alpha = 0°$.

2. Second-Order Antisymmetric Load Distributions $\overline{\Delta p}_{s2}^{(as)}$ and $\overline{\Delta p}_{f2}$

Substituting Eqs. (102–104) into Eqs. (45) and (47) respectively, and noting that $H_0 = 0$ in this case, we obtain the normal wash conditions as follows:

$$\bar{W}_2^{(as)}(x', y^*)/(U_0\Psi_0\alpha) = -ikS_s y^* + \frac{1}{\Psi_0}\frac{\partial\bar{\phi}_1^{(as)**}}{\partial x'} \tag{106}$$
$$\bar{V}_2(x', z^*)/(U_0\Psi_0\alpha) = 0$$

With these normal wash conditions, Eqs. (75) and (76) are solved for the unknown coefficients $a_{s,nm}^{(as)(2)}$ and $a_{f,nm}^{(2)}$. A converged solution was obtained by using 72 control points in total (5 chordwise and 8 spanwise on the tailplane, and 4 chordwise and 8 spanwise on the fin). The chordwise locations of the control points on the tailplane were 5%, 35%, 62.5%, 87.5% and 99.95% chord points from the leading edge and the spanwise locations were 11%, 22%, 33%, 44%, 55%, 66%, 77% and 88% semispan points from the midspan. The chordwise locations of the control points on the fin were 25%, 75%, 90%, and 99.95% chord points from the leading edge. The same spanwise locations as those used for the tailplane were used for the fin.

The distributions of $\overline{\Delta p}_{s2}^{(as)*}$ and $\overline{\Delta p}_{f2}^*$ are shown in Fig. 2. As already discussed in Sec. IV. 1), $\overline{\Delta p}_{s2}^{(as)*}$ and $\overline{\Delta p}_{f2}^*$ remain finite at the trailing edges. The true second-order antisymmetric load distributions of $\overline{\Delta p}_{s2}^{(as)}$ on the tailplane for $k = 0.25$ calculated by the use of Eq. (62) are shown in Fig. 3. The second-order rolling moments acting on the tailplane calculated by integrating $\Delta p_{s2}^{(as)}$ are plotted against the reduced frequency in Fig. 4. To show the effect of the fin, the rolling moments for the isolated tailplane calculated by the method of Ref. 10 are also shown in Fig. 4 by the dotted line. The considerable effect of the fin on the rolling moments is seen for both amplitudes and phase angles. This effect is mainly caused by the existence of the first-order induced velocity terms, particularly $\partial\bar{\phi}_{s1}^{(as)**}/\partial x'$, in the expression of $\bar{\Phi}_s^{(as)}$ [Eq. (40)]. Such terms do not appear in the case of an isolated wing [Eq. (28) of Ref. 10].

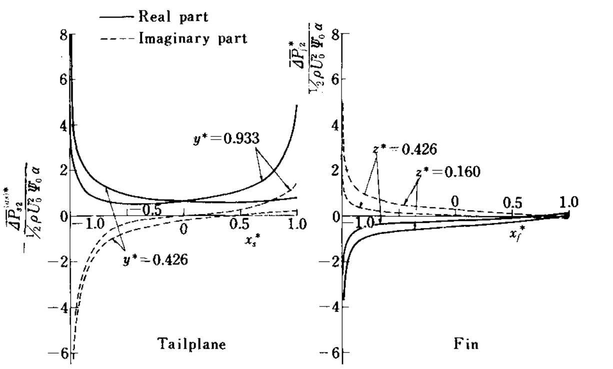

Fig. 2 $k = 0.25$. **Distributions of $\overline{\Delta p}_{s2}^{(as)*}$ and $\overline{\Delta p}_{f2}^*$.**

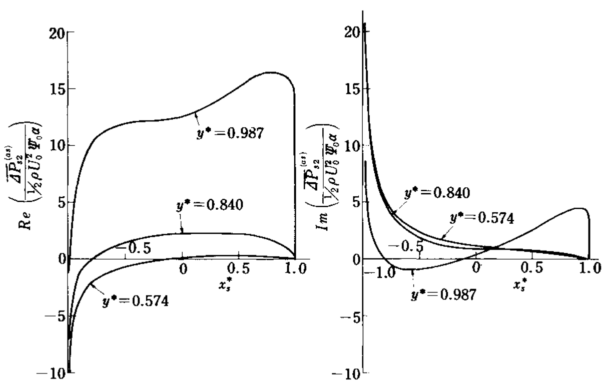

Fig. 3 $k = 0.25$. **Second-order antisymmetric load distributions on tailplane of simplified T-tail.**

** The first-order rolling moments are independent of α.

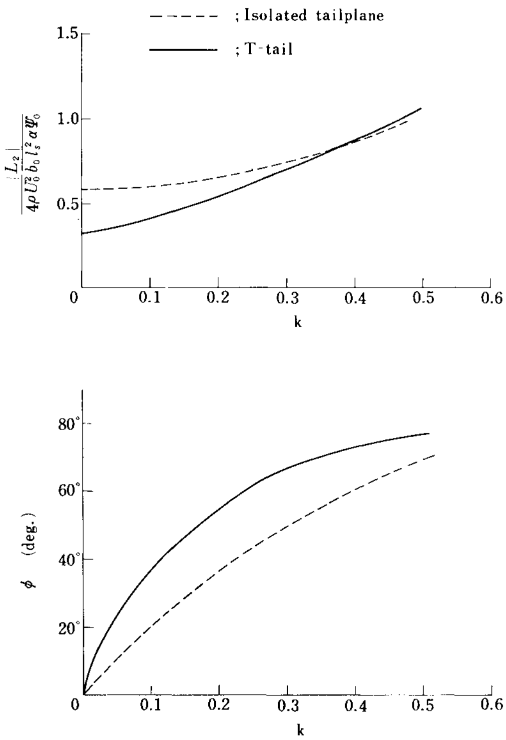

Fig. 4 Comparison of second-order rolling moments of tailplane with and without fin.

Using the value of $\alpha = 5°29'$, the total amount of the rolling moments acting on the tailplane, which is the sum of the first- and second-order rolling moments, are calculated and plotted by the solid line in Fig. 5. They are compared with the experimental values[11] obtained at $\alpha = 5°29'$. The agreement is satisfactory both in phase angle and in absolute value.

As a second example we consider another T-tail whose plan form is as follows: The tailplane is 30° swept back of constant chord, and of aspect ratio 3. The fin is rectangular and of aspect ratio 3. The axis of yaw is through the midchord of the fin. The angle of attack of the tailplane is 5° 2′. The same locations of the control points as those of the first example are used in this case also. The first-order rolling moments acting on the tailplane is calculated and plotted by the dotted line in Fig. 6, being compared with the experimental values obtained at $\alpha \doteqdot 0°$. Using the value of $\alpha = 5°2'$, the total amount of the rolling moments acting on the tailplane, which is the sum of the first- and second-order rolling moments, are calculated and plotted by the solid line in Fig. 6. They are compared with the experimental values[11] obtained at $\alpha = 5°2'$.

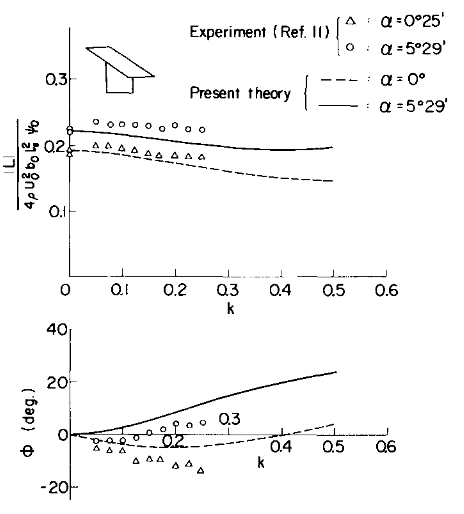

Fig. 5 Rolling moment of horizontal tailplane for simplified T-tail oscillating in yaw about fin midchord ($A_s = 3$, $\Lambda_s = 0°$, $\lambda_s = 1$, $\Gamma = 0°$, $A_f = 3$, $\Lambda_f = 0°$, $\lambda_f = 1$).

VII. Conclusion

A lifting-surface theory for an oscillating T-tail with the tailplane at incidence has been developed. The integral equations for predicting the antisymmetric load distributions induced by the in-plane motion of the tailplane have been derived from the second-order boundary-value problem, while the first-order problems are those of conventional lifting-surface theories. A procedure for solving the integral equations has been proposed. The rolling moments acting on the tailplane of the simplified T-tails oscillating in yaw were calculated and compared with the experimental values. The agreement is satisfactory both in absolute value and phase angle. The theory will enable more rigorous flutter analysis of T-tails.

Appendix

$$K_{ss}(k, x'-\xi', y'-\eta') = \frac{e^{-ik(x'-\xi')}}{b_0{}^2(y'-\eta')^2}\Big\{-ik|y'-\eta'| + k|y'-\eta'|K_1(k|y'-\eta'|) + \frac{i\pi}{2}k|y'-\eta'| \times [I_1(k|y'-\eta'|) - L_1(k|y'-\eta'|)] + \frac{(x'-\xi')e^{ik(x'-\xi')}}{\{(x'-\xi')^2+(y'-\eta')^2\}^{1/2}} - ik|y'-\eta'|\int_0^{(x'-\xi')/|y'-\eta'|}\frac{\tau}{(1+\tau^2)^{1/2}}e^{ik|y'-\eta'|\tau}d\tau\Big\} \qquad (A1)^5$$

$$K_{sf}(k, x'-\xi', y', \zeta') = \frac{e^{-ik(x'-\xi')}}{b_0{}^2}\Big\{k^2\frac{\zeta' y'}{y'^2+\zeta'^2} \times \Big[K_2[k(y'^2+\zeta'^2)^{1/2}] - \frac{i\pi}{2}\Big(I_0[k(y'^2+\zeta'^2)^{1/2}] - L_0[k(y'^2+\zeta'^2)^{1/2}] - \frac{2}{k(y'^2+\zeta'^2)^{1/2}}\{I_1[k(y'^2+\zeta'^2)^{1/2}] - L_1[k(y'^2+\zeta'^2)^{1/2}]\} + \frac{2}{\pi k(y'^2+\zeta'^2)^{1/2}}\Big)\Big] + \frac{(x'-\xi')\zeta' y' e^{ik(x'-\xi')}}{(y'^2+\zeta'^2)[(x'-\xi')^2+y'^2+\zeta'^2]^{1/2}}\Big(\frac{2-ik(x'-\xi')}{y'^2+\zeta'^2} + \frac{1}{(x'-\xi')^2+y'^2+\zeta'^2}\Big) - \frac{ik\zeta' y'}{y'^2+\zeta'^2}\Big(\frac{1}{(y'^2+\zeta'^2)^{1/2}} \times \int_0^{(x'-\xi')/(y'^2+\zeta'^2)^{1/2}}\frac{\tau}{(1+\tau^2)^{1/2}}e^{ik(y'^2+\zeta'^2)^{1/2}\tau}d\tau - ik\int_0^{(x'-\xi')/(y'^2+\zeta'^2)^{1/2}}\frac{\tau^2}{(1+\tau^2)^{1/2}}e^{ik(y'^2+\zeta'^2)^{1/2}\tau}d\tau\Big)\Big\} \qquad (A2)$$

$$K_{ff}(k, x'-\xi', z'-\zeta') = K_{ss}(k, x'-\xi', z'-\zeta') \qquad (A3)$$

$$K_{fs}(k, x'-\xi', \eta', z') = K_{sf}(k, x'-\xi', \eta', z') \qquad (A4)$$

The kernel functions necessary for evaluating the first-order induced velocities are as follows:

$$K_{sx}(k, x'-\xi', y', \zeta') = \frac{1}{b_0{}^2}\Big[\!\Big[\frac{y'}{[(x'-\xi')^2+y'^2+\zeta'^2]^{3/2}} + \frac{iky' e^{-ik(x'-\xi')}}{[(x'-\xi')^2+y'^2+\zeta'^2]^{1/2}}\Big\{k\Big[-K_1[k(y'^2+\zeta'^2)^{1/2}] - \frac{i\pi}{2}\Big(I_1[k(y'^2+\zeta'^2)^{1/2}] - L_1[k(y'^2+\zeta'^2)^{1/2}] - \frac{2}{\pi}\Big)\Big] - \frac{(x'-\xi')e^{ik(x'-\xi')}}{(y'^2+\zeta'^2)^{1/2}[(x'-\xi')^2+y'^2+\zeta'^2]^{1/2}} + ik\int_0^{(x'-\xi')/(y'^2+\zeta'^2)^{1/2}}\frac{\tau}{(1+\tau^2)^{1/2}}e^{ik(y'^2+\zeta'^2)^{1/2}\tau}d\tau\Big\}\Big]\!\Big] \qquad (A5)$$

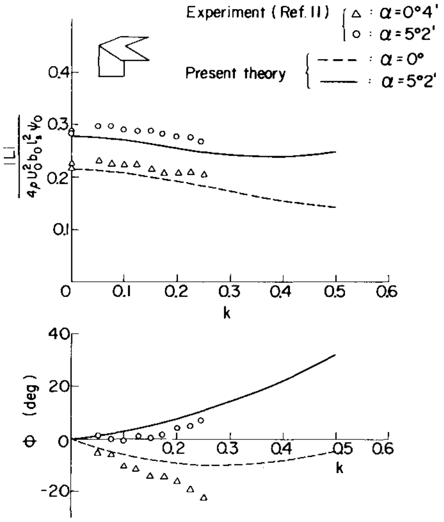

Fig. 6 Rolling moment of horizontal tailplane for simplified T-tail oscillating in yaw about fin midchord ($A_s = 3$, $\Lambda_s = 30°$, $\lambda_s = 1$, $\Gamma = 0°$, $A_f = 3$, $\Lambda_f = 0°$, $\lambda_f = 1$).

$$
\begin{aligned}
K_{sy}(k, x'-\xi', y', \zeta') = {} & \frac{e^{-ik(x'-\xi')}}{b_0{}^2}\left\{\frac{k}{(y'^2+\zeta'^2)^{1/2}} \times \right.\\
& \left[K_1[k(y'^2+\zeta'^2)^{1/2}] + \frac{i\pi}{2}\left(I_1[k(y'^2+\zeta'^2)^{1/2}] - \right.\right.\\
& \left.\left. L_1[k(y'^2+\zeta'^2)^{1/2}] - \frac{2}{\pi}\right)\right] - \frac{k^2y'^2}{y'^2+\zeta'^2}\left[K_2[k(y'^2+\zeta'^2)^{1/2}] - \right.\\
& \frac{i\pi}{2}\left(I_0[k(y'^2+\zeta'^2)^{1/2}] - L_0[k(y'^2+\zeta'^2)^{1/2}] - \right.\\
& \frac{2}{k(y'^2+\zeta'^2)^{1/2}}\{I_1[k(y'^2+\zeta'^2)^{1/2}] - L_1[k(y'^2+\zeta'^2)^{1/2}]\} + \\
& \left.\left.\frac{2}{\pi k(y'^2+\zeta'^2)^{1/2}}\right)\right] + \frac{(x'-\xi')e^{ik(x'-\xi')}}{(y'^2+\zeta'^2)[(x'-\xi')^2+y'^2+\zeta'^2]^{1/2}} \times \\
& \left(1 - 2\frac{y'^2}{y'^2+\zeta'^2} - \frac{y'^2}{(x'-\xi')^2+y'^2+\zeta'^2}\right) - i\frac{k}{(y'^2+\zeta'^2)^{1/2}} \times \\
& \left(1 - \frac{y'^2}{y'^2+\zeta'^2}\right)\int_0^{(x'-\xi')/(y'^2+\zeta'^2)^{1/2}} \frac{\tau}{(1+\tau^2)^{1/2}} \times \\
& e^{ik(y'^2+\zeta'^2)^{1/2}\tau}\,d\tau + ik\frac{(x'-\xi')^2y'^2}{(y'^2+\zeta'^2)^2}\frac{e^{ik(x'-\xi')}}{[(x'-\xi')^2+y'^2+\zeta'^2]^{1/2}} + \\
& \left. k^2\frac{y'^2}{y'^2+\zeta'^2}\int_0^{(x'-\xi')/(y'^2+\zeta'^2)^{1/2}} \frac{\tau^2}{(1+\tau^2)^{1/2}} e^{ik(y'^2+\zeta'^2)^{1/2}\tau}\,d\tau\right\}
\end{aligned}
\qquad \text{(A6)}
$$

$$
K_{fx}(0, x'-\xi', \eta', z') = \frac{z'}{b_0{}^2[(x'-\xi')^2+\eta'^2+z'^2]^{3/2}} \qquad \text{(A7)}
$$

$$
\begin{aligned}
K_{fz}(0, x'-\xi', \eta', z') = {} & \frac{1}{b_0{}^2(\eta'^2+z'^2)}\left[1 - \frac{2z'^2}{\eta'^2+z'^2} + \right.\\
& \frac{x'-\xi'}{[(x'-\xi')^2+\eta'^2+z'^2]^{1/2}}\left(1 - \frac{2z'^2}{\eta'^2+z'^2} - \right.\\
& \left.\left.\frac{z'^2}{(x'-\xi')^2+\eta'^2+z'^2}\right)\right]
\end{aligned}
\qquad \text{(A8)}
$$

In the above I_0 and I_1 are modified Bessel functions of the first kind, K_1 and K_2 are modified Bessel functions of the second kind and L_0 and L_1 are Strube functions.

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